

### Worked Solutions

#### Edexcel C3 Paper F

1. (a)  $\frac{2}{x+3} - \frac{1}{(x+3)(x+4)}$

$$\frac{2(x+4) - 1}{(x+3)(x+4)}$$

$$\frac{2x+7}{(x+3)(x+4)}$$

(4)

(b)  $2x+7=0 \Rightarrow x = -3\frac{1}{2}$

(2)

2. (a)  $f^{-1}: x \mapsto \frac{2}{x} + 3, \quad x \in \mathbb{R}, \quad x \neq 0$

let  $y = \frac{2}{x-3}$

$$yx - 3y = 2$$

$$yx = 2 + 3y$$

$$x = \frac{2}{y} + 3$$

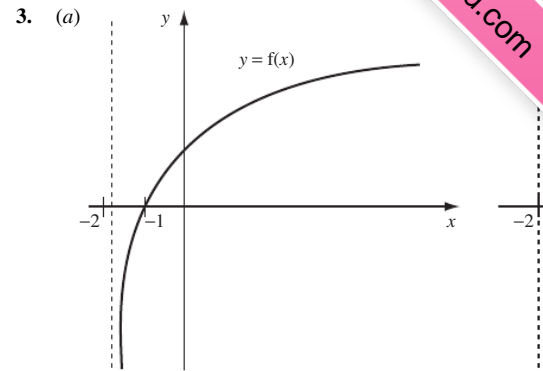
$$\therefore f^{-1}: x \mapsto \frac{2}{x} + 3, \quad x \in \mathbb{R}, \quad x \neq 0$$

(4)

(b)  $f(4) = \frac{2}{4-3} = 2$

$$ff^{-1}(7) = 7$$

(2)



(b) The line  $y = x$  only cuts the graph once.

(c)  $\left. \begin{array}{l} \ln(2+2) - 2 = -0.6137 \\ \ln(1+2) - 1 = 0.098 \end{array} \right\} \text{change of sign}$

(d)  $x_1 = 1.0986, x_2 = 1.1309, x_3 = 1.1413, x_4 = 1.146$   
solution 1.146 (3 d.p.).

4. (a)  $x^2 \cdot \frac{1}{x} + 2x \ln x = x(1 + 2 \ln x)$

(b)  $2 \cos 3x \cdot (-\sin 3x) \cdot 3 = -6 \sin 3x \cos 3x$

(c)  $\frac{x \cdot \cos x - \sin x}{x^2}$

5. (a) R.H.S =  $\frac{1 - \tan^2 \theta}{2 \tan \theta}$  ( $\times \tan^2 \theta$ )

$$\equiv \frac{1}{\tan 2\theta}$$

$$\equiv \cot 2\theta$$

(5)

(b)  $\cot^2 \theta - 1 = 2 \cot \theta$

$$\therefore \cot 2\theta = 1 \Rightarrow \tan 2\theta = 1$$

$$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\therefore \theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

(4)

6. (a)  $e^{2x} + 6 = 5e^x$

$$e^{2x} - 5e^x + 6 = 0$$

$$(e^x - 3)(e^x - 2) = 0$$

(3)

(b)  $e^x = 3 \Rightarrow x = \ln 3$  (or 1.0986 4 d.p.)

and  $e^x = 2$

$$\Rightarrow x = \ln 2$$
 (or 0.6931 to 4 d.p.)

(4)

(c)  $e^{2(x+1)} - 5e^{x+1} + 6 = 0$

$$\left. \begin{array}{l} e^{x+1} = 3 \\ e^{x+1} = 2 \end{array} \right\} \Rightarrow \begin{array}{l} x = \ln 3 - 1 \quad (\text{or } 0.0986 \quad 4 \text{ d.p.}) \\ x = \ln 2 - 1 \quad (\text{or } -0.3069 \quad 4 \text{ d.p.}) \end{array}$$

(4)

7. (a)  $25 \left( \frac{7}{25} \sin x + \frac{24}{25} \cos x \right) \equiv R(\cos \alpha \dots)$

$$\Rightarrow R = 25 \text{ and } \tan \alpha = \frac{24}{7} \Rightarrow \alpha = 73.7^\circ$$

(b)  $25 \sin(x + 73.7) = 15$

$$\sin(x + 73.7) = \frac{3}{5}$$

$$x + 73.7 = 36.9, 143.1, 360 + 36.9$$

$$x = 69.4^\circ, 323.2^\circ$$

(c)  $15 \sec x - 7 \tan x = 24$

$$\Rightarrow 15 - 7 \sin x = 24 \cos x.$$

(d)  $25, x = 16.3$

8. (a)  $\frac{dx}{dy} = \cos y$

(b)  $y = \frac{\pi}{4}$   $\cos y = \frac{1}{\sqrt{2}}$   $\therefore$  grad. of tgt =  $\sqrt{2}$

(c)  $y - \frac{\pi}{4} = \sqrt{2} \left( x - \frac{1}{\sqrt{2}} \right)$

(d)  $y = 0, -\frac{\pi}{4} = \sqrt{2}x - 1$

$$\sqrt{2}x = 1 - \frac{\pi}{4}$$

$$x = \frac{1 - \frac{\pi}{4}}{\sqrt{2}} = \frac{4 - \pi}{4\sqrt{2}}$$

(e) area =  $\left( \frac{4 - \pi}{4\sqrt{2}} \right) \cdot \frac{\pi}{8}$